Active control of beam structures with piezoelectric actuators and sensors: modeling and simulation

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TECHNICAL NOTE

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Abstract

The active damping of structures is an important emerging field. In this context, it is necessary to be able to develop new control methods for flexible structures and simulate their effects. In order to be able to deal with the optimization of active device locations, spillover and any other general problems linked to control and model reduction, a simple but sufficiently rich model is very useful. This is the reason why this technical note deals with the modeling and simulation of the active vibration control of beam structures using piezoelectric actuators and sensors.

In order to model beam structures equipped with piezoelectric devices, we develop a simple finite composite beam element, taking into account the properties of piezoelectric elements. This model uses six mechanical degrees of freedom and four electric degrees of freedom. Then, a linear quadratic regulator method is used to compute the control, including the implementation of a state observer. Several simulations are presented.

1. Introduction

This technical note deals with the modeling of beam structures equipped with piezoelectric devices (actuators and sensors). In the case of a simple beam equipped with one piezoelectric actuator and one sensor an analytical study can easily be developed; but, in cases of structures made with several beams equipped with actuators and sensors, a discrete model is necessary in order to take into account the piezoelectric effect. Moreover, most of piezoelectric finite elements are three dimensional or two dimensional [1–4]. The use of these finite models in the case of beam structures is not optimal, in particular to solve structural optimization problems which are computationally expensive. Finally, the active damping of structures is an important emerging field. In this context, it is necessary to be able to develop new control methods for flexible structures and simulate their effects. In order to be able to deal with the optimization of active device locations, spillover and any other general problems linked to control and model reduction, a simple but sufficiently rich model is very useful.

For this purpose, a simple finite beam element is described. It corresponds to a composite beam made up of three layers: one elastic and two piezoelectric. In this study, to simplify the presentation, we consider a 2D beam finite element, but all developments can be generalized to a 3D beam finite element. Six mechanical degrees of freedom and four electric degrees of freedom are used. From variational principles, the generalized discrete equations are obtained. In order to set up the active control the second differential equations are transformed into a state space model. Then, principles of control theory can be used: a linear quadratic control method, including a state observer, is considered. Several simulations are presented. The first one shows active damping of a simple cantilever beam using one piezoelectric actuator and one sensor. It gives a first validation of the finite element beam. Others simulations consider a flexible three-beam structure.

2. Finite element formulation

Consider a flexible elastic beam structure, as shown in figure 1, controlled by several piezoelectric actuators and sensors. Each device is made up of a pair of piezoelectric materials, polarized in the thickness direction and attached symmetrically. The top and bottom sides of each piezoelectric are covered by

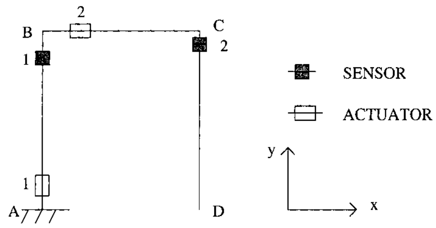

Figure 1. A three-beam structure.

electrodes to ensure the connection with the electric circuit. At the device locations, the structure is a composite in its thickness direction: it is made up of two piezoelectric layers bonded onto an elastic layer. Consequently, we consider a composite beam $[A, B]$ composed by two piezoelectric materials and one elastic.

Assuming that the structure is composed by long beams, Euler's beam theory is used. The displacement in a cross section of each beam of the structure is written as

$$\vec{U}(x, y, z, t) = (u(x, t) + \theta(x, t)z)\vec{x} + v(x, t)\vec{z}$$

where $\theta(x, t) = (\partial v/\partial x)(x, t)$, $\vec{x}$ is the local axis of the beam and $\vec{z}$ the local axis in the beam thickness direction. The element nodal displacement variable $\{U^e\}$ is then defined as (figure 2)

$$\{U^e(t)\} = \{\, u_A \quad v_A \quad \theta_A \quad u_B \quad v_B \quad \theta_B \,\}^T$$

where $(u_A,\ v_A,\ \theta_A)$ and $(u_B,\ v_B,\ \theta_B)$ are the longitudinal displacements, the normal displacements and the rotations about the y-axis at nodes A and B. The generalized displacements in the element can be expressed in nodal variables by finite element interpolation functions as follows:

$$\begin{aligned}\{U^h\}(x, t) &= \{\, u^h(x, t) \quad v^h(x, t) \quad \theta_y^h(x, t) \,\}^T \\ &= [N^e(x)]\{U^e(t)\}\end{aligned}$$

where $[N^e]$ is the displacement shape functions matrix given by

$$\begin{pmatrix} 1 - x & 0 & 0 & x & 0 & 0 \\ 0 & 1 - 3x^2 + 2x^3 & L_e(x - 2x^2 + x^3) & 0 & 3x^2 - 2x^3 & L_e(-x^2 + x^3) \\ 0 & \dfrac{1}{L_e}(-6x + 6x^2) & 1 - 4x + 3x^2 & 0 & \dfrac{1}{L_e}(6x - 6x^2) & -2x + 3x^2 \end{pmatrix}$$

L_e is the length of the finite element. The strain–displacement is obtained using the strain differential operator:

$$\begin{aligned}\{\partial U^h\}(x, t) &= \left(\frac{\partial u^h}{\partial x} \quad \frac{\partial v^h}{\partial x} \quad \frac{\partial \theta_y^h}{\partial x} \right)^T (x, t) \\ &= [B_U^e]\{U^e(t)\}.\end{aligned}$$

For electric variables, as the thickness of piezoelectric parts is small, we assume that the electric potential Φ is constant on each electrode and that the electric field is constant in the piezoelectric; then they are directly associated with the element and the element nodal electric potential variable $\{\Phi^e\}$ is defined as (figure 2)

$$\{\Phi^e\} = \{\, \Phi_1 \quad \Phi_2 \quad \Phi_3 \quad \Phi_4 \,\}^T$$

where Φ_1, Φ_2, Φ_3, Φ_4 represent the electric potentials on each electrode of the element. These four element potential variables allow us to consider many possible connections (in order to excite bending or longitudinal motions) by coupling in several ways the actuators or sensors. As the electric field is constant, it is given by

$$\begin{aligned}\{E^h\} = \begin{pmatrix} E^{(1)} \\ E^{(2)} \end{pmatrix} &= \begin{pmatrix} -\dfrac{1}{h_1} & \dfrac{1}{h_1} & 0 & 0 \\ 0 & 0 & -\dfrac{1}{h_2} & \dfrac{1}{h_2} \end{pmatrix} \{\Phi^e\} \\ &= [B_\Phi^e]\{\Phi^e\}\end{aligned}$$

where $E^{(1)}$, h_1 and $E^{(2)}$, h_2 are the electric fields in and the thicknesses of piezoelectric parts 1 and 2.

Then, assuming that no electric charge is applied to the piezoelectrics, variational principles give the following element equations [5]:

$$[K_{UU}^e]\{U^e\} + [K_{U\Phi}^e]\{\Phi^e\} + [M_{UU}^e]\{\ddot{U}^e\} = \{F_U^e\} \qquad (1)$$

$$[K_{\Phi U}^e]\{U^e\} + [K_{\Phi\Phi}^e]\{\Phi^e\} = \{0\} \qquad (2)$$

where $[K_{UU}^e]$, $[M_{UU}^e]$ and $\{F_U^e\}$ are the element stiffness matrix, the element mass matrix and the applied load vector. These matrices contain homogenized mechanical coefficients according to the beam section. Dots indicate a derivative with respect to time. $[K_{U\Phi}^e]$ and $[K_{\Phi U}^e]$ couple the mechanical properties to the electric properties and $[K_{\Phi\Phi}^e]$ is the electric stiffness matrix. For the piezoelectric layer i we define the following constants:

$$S^{(i)} = \int\int_{A^{(i)}} z \,\mathrm{d}y\,\mathrm{d}z \qquad \Gamma_{U\Phi}^{(i)} = \frac{d_{31}^{(i)} E^{(i)} S^{(i)}}{h^{(i)}}$$

$$\Gamma_{\Phi U}^{(i)} = \frac{e_{311}^{(i)} S^{(i)}}{h^{(i)}} \qquad \Pi_{U\Phi}^{(i)} = \frac{d_{31}^{(i)} E^{(i)} A^{(i)}}{h^{(i)}}$$

$$\Pi_{\Phi U}^{(i)} = \frac{e_{311}^{(i)} A^{(i)}}{h^{(i)}} \qquad \Pi_{\Phi\Phi}^{(i)} = \frac{\epsilon_{33}^{(i)} A^{(i)} L_e}{(h^{(i)})^2}$$

where $d_{31}^{(i)}$, $e_{311}^{(i)}$, $\epsilon_{33}^{(i)}$, $E^{(i)}$, $h^{(i)}$ and $A^{(i)}$ are the piezoelectric constants, Young's modulus, the thickness and the section of the piezoelectric layer, the expressions of $[K_{U\Phi}^e]$, $[K_{\Phi U}^e]$ and $[K_{\Phi\Phi}^e]$ are:

$$[K_{U\Phi}^e] = \begin{pmatrix} -\Pi_{U\Phi}^{(1)} & \Pi_{U\Phi}^{(1)} & -\Pi_{U\Phi}^{(2)} & \Pi_{U\Phi}^{(2)} \\ 0 & 0 & 0 & 0 \\ \Gamma_{U\Phi}^{(1)} & -\Gamma_{U\Phi}^{(1)} & \Gamma_{U\Phi}^{(2)} & -\Gamma_{U\Phi}^{(2)} \\ \Pi_{U\Phi}^{(1)} & -\Pi_{U\Phi}^{(1)} & \Pi_{U\Phi}^{(2)} & -\Pi_{U\Phi}^{(2)} \\ 0 & 0 & 0 & 0 \\ -\Gamma_{U\Phi}^{(1)} & \Gamma_{U\Phi}^{(1)} & -\Gamma_{U\Phi}^{(2)} & \Gamma_{U\Phi}^{(2)} \end{pmatrix}$$

$$[K_{\Phi U}^e] = \begin{pmatrix} \Pi_{\Phi U}^{(1)} & 0 & -\Gamma_{\Phi U}^{(1)} & -\Pi_{\Phi U}^{(1)} & 0 & \Gamma_{\Phi U}^{(1)} \\ -\Pi_{\Phi U}^{(1)} & 0 & \Gamma_{\Phi U}^{(1)} & \Pi_{\Phi U}^{(1)} & 0 & -\Gamma_{\Phi U}^{(1)} \\ \Pi_{\Phi U}^{(2)} & 0 & -\Gamma_{\Phi U}^{(2)} & -\Pi_{\Phi U}^{(2)} & 0 & \Gamma_{\Phi U}^{(2)} \\ -\Pi_{\Phi U}^{(2)} & 0 & \Gamma_{\Phi U}^{(2)} & \Pi_{\Phi U}^{(2)} & 0 & -\Gamma_{\Phi U}^{(2)} \end{pmatrix}$$

$$[K_{\Phi\Phi}^e] = \begin{pmatrix} \Pi_{\Phi\Phi}^{(1)} & -\Pi_{\Phi\Phi}^{(1)} & 0 & 0 \\ -\Pi_{\Phi\Phi}^{(1)} & \Pi_{\Phi\Phi}^{(1)} & 0 & 0 \\ 0 & 0 & \Pi_{\Phi\Phi}^{(2)} & -\Pi_{\Phi\Phi}^{(2)} \\ 0 & 0 & -\Pi_{\Phi\Phi}^{(2)} & \Pi_{\Phi\Phi}^{(2)} \end{pmatrix}.$$

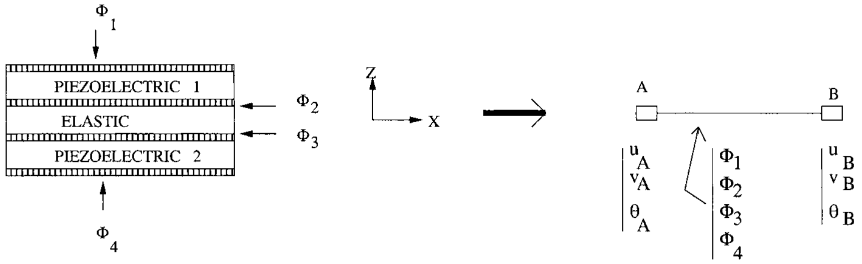

Figure 2. Modeling of a composite beam.

The analytical expressions of $[M^e_{UU}]$ and $[K^e_{UU}]$ are detailed in [6, 7].

Then, the assembled form of (1) and (2), for a beam structure equipped with N_a actuators and N_c sensors can be written as

$$[K_{UU}]\{q_U\} + [K_{U\Phi}]_a\{q_\Phi\}_a + [M_{UU}]\{\ddot{q}_U\} = \{F_U\} \quad (3)$$

$$[K_{\Phi U}]_s\{q_U\} + [K_{\Phi\Phi}]_s\{q_\Phi\}_s = \{0\} \quad (4)$$

where $\{q_U\}$ (size $Nddl$), $\{q_\Phi\}_s$ (size $\bar{N}_s$) and $\{q_\Phi\}_a$ (size $\bar{N}_a$) are the generalized displacements and the generalized potentials of the sensors and the actuators. $\bar{N}_s$ and $\bar{N}_a$ are the number of the unknown potentials of the sensors and actuators. They are such that $\bar{N}_s \leqslant 4N_s$ and $\bar{N}_a \leqslant 4N_a$. $[K_{UU}]$, $[K_{U\Phi}]_a$, $[M_{UU}]$, $[K_{\Phi U}]_s$, $[K_{\Phi\Phi}]$ and $\{F_U\}$ are the generalized discrete matrices. In addition to these two equations, initial conditions have to be added.

In order to set up a control law damping the vibrations caused by external disturbances $\{F_U\}$ or by the initial conditions, a state space model is developed in the next section and a linear quadratic regulator (LQR) method, including a state observer, is used.

3. The control system

The application of the active control methods in a dynamic structural problem requires the use of a state space model. To obtain this kind of equation, the solution $\{q_U\}$ is decomposed into the normalized orthogonal modal basis $\{\Psi_n\}$. Assuming that the contribution of the highest modes is negligible, we keep only the first N eigenfunctions:

$$\{q_U\} = \sum_{r=1}^{N}\{\Psi_r\}\alpha_r(t) = [\Psi]_{(Nddl,N)}\{\alpha\}_{(N,1)}. \quad (5)$$

Substituting this equation into (3) and (4), and using the orthogonality properties of modes leads to the state equations:

$$\frac{\mathrm{d}\{x\}}{\mathrm{d}t} = [A]\{x\} + [B]\{q_\Phi\}_a + \{g\} \quad (6)$$

$$\{x\}(t=0) = \{x_0\} \quad (7)$$

and

$$\{y\} = [C]\{x\} \quad (8)$$

where the normalized state vector (size $2N$) is

$$\{x\} = \{\,\omega_n\alpha_n \quad \dot{\alpha}_n\,\}^T \quad (9)$$

$[A]$, $[B]$, $[C]$ and $\{g\}$ are the state, control output and load matrices, given by:

$$[A] = \begin{pmatrix} [0] & [\omega] \\ [-\omega] & -2[\delta][\omega] \end{pmatrix} \qquad [B] = \begin{pmatrix} [0] \\ [\Psi]^T[K_{U\Phi}]_a \end{pmatrix}$$

$$[C] = \begin{pmatrix} \dfrac{-[K_{\Phi\Phi}]_s^{-1}[K_{\Phi U}]_s\{\Psi_r\}}{\omega_r} & [0] \end{pmatrix}$$

$$\{g\} = \begin{pmatrix} \{0\} \\ [\Psi]^T\{F\} \end{pmatrix}$$

$\{x_0\}$ is the initial conditions vector. As usual, a term of modal viscous damping has been added to take into account a small amount of damping. $[\delta]$ is the diagonal matrix of the damping ratio and $[\omega]$ is the diagonal matrix containing the natural angular frequencies.

In order to obtain a controlled system having good stability and robustness, we chose the LQR control method [8]. Assuming that the state equation is controllable, it led us to use the control law

$$\{q_\Phi\}_a = -[K]\{x\} \quad (10)$$

which minimizes a cost function given by

$$J_\Phi = \frac{1}{2}\int_0^\infty [\{x\}^T[Q]\{x\} + \{q_\Phi\}_a^T[R]\{q_\Phi\}_a]\,\mathrm{d}t \quad (11)$$

where $[R]$ is a positive matrix and $[Q]$ is a positive semidefinite matrix. The choice of $[Q]$ and $[R]$ is not easy. In the following applications, $[Q]$ is chosen so that $\{x\}^T[Q]\{x\}$ represents the mechanical energy. $[R]$ is a diagonal matrix, the components of which are chosen such that the maximal values of $\{q_\Phi\}_a$ are less than the maximal admissible values for the piezoelectric materials under consideration. In order to be implemented, the optimal state control law obviously needs knowledge of the state vector $\{x\}$. This knowledge is not complete since only the output voltages in $\{y\}$ are observed. Assuming that the state system verifies the observability criteria, an estimation $\{\hat{x}\}$ is computed using a Luenberger observer [8, 9]. Consequently, the control law applied to the actuators becomes

$$\{q_\Phi\}_a = -[K]\{\hat{x}\}.$$

4. Applications

In this section, we present two applications. In each case the structure is equipped with devices made with the same piezoelectric material (figure 2). In order to limit the study to bending motions, for each actuator and sensor we set $\Phi_2 = \Phi_3 = 0$, and $\Phi_1 = -\Phi_4$ (phase opposition between the

Table 1. Characteristics of the simple cantilever beam.

Length of the beam (m)	1
Length of the actuator and the sensor (m)	0.06
Width (m)	0.02
Thickness (m)	0.002
Mass density (kg m^{-3})	2700
Young's modulus (Pa)	7×10^{10}
Natural frequencies (Hz)	1.64, 10.29, 28.81, 56.46
Damping ratio	$\simeq$0.1%

Table 2. Characteristics of piezoelectric PZT.

Width (m)	0.01
Thickness (m)	0.001
Mass density (kg m^{-3})	7440
Young's modulus (Pa)	4×10^{10}
Piezoelectric constant ϵ_{33}	1.72×10^{-8}
Piezoelectric constant d_{31} (m V^{-1})	230×10^{-12}
Maximal admissible voltage (V)	250

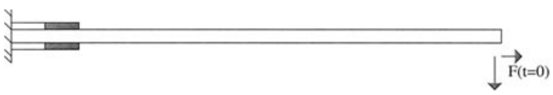

Figure 3. A simple cantilever beam.

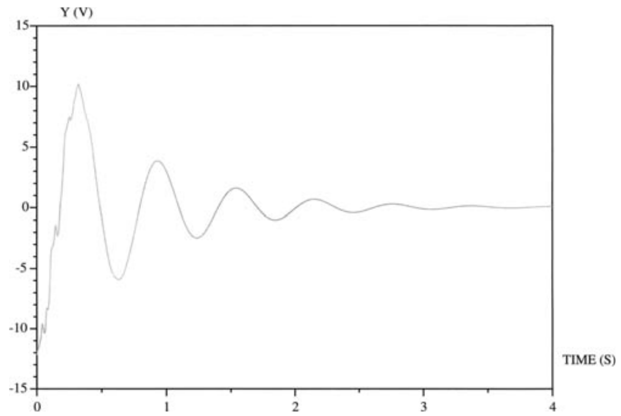

Figure 4. Comparison of the analytical calculus (black line) and discrete model (grey line) for the sensor output.

two piezoelectric parts of each element). Consequently, here we only consider one electric variable for each device. The finite element was implemented in DYNADID2D [10]. The construction of the control and the observer was done using SCILAB [11].

First, we study the active control of a simple cantilever beam in the case of a release test, equipped with one piezoelectric actuator and one sensor located near the fixed edge (figure 3). The geometrical and mechanical characteristics of the system are detailed in tables 1 and 2. The initial conditions are derived from an initial load $\vec{F}(t = 0) = 0.003\vec{z}$ applied to the free end of the beam.

Because of the nature of the excitation, we take into account only the first four eigenmodes. The study of this simple structure can give us a first validation of the finite beam element. The idea is to compare the results obtained using the finite element discretization with analytical results [5]. For this purpose the mechanical characteristics of piezoelectric

Table 3. Characteristics of the three-beam structure.

B_1 (m)	(0, 0)
Length B_1B_2 (m)	0.5
Length B_2B_3 (m)	0.4
Length B_3B_4 (m)	0.5
Location of actuator 1 (m)	(0, 0.02)
Location of actuator 2 (m)	(0.04, 0.5)
Location of sensor 1 (m)	(0, 0.42)
Location of sensor 2 (m)	(0.4, 0.46)
Length of each actuator (m)	0.06
Length of each sensor (m)	0.01
Natural frequencies (Hz)	1.48, 2.89, 7.99, 29.64
Width of elastic beams (m)	0.025
Thickness of elastic beams (m)	0.002
Mass density of elastic beams (kg m^{-3})	2700
Young's modulus of elastic beams (Pa)	7.3×10^{10}
Damping ratio	$\simeq$0.025%

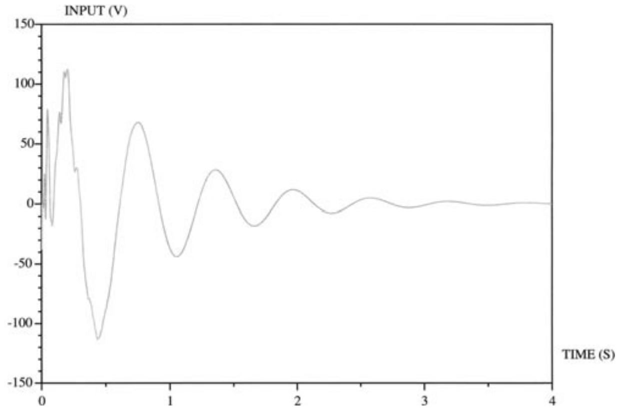

Figure 5. Comparison of the analytical calculus (black line) and discrete model (grey line) for the actuator output.

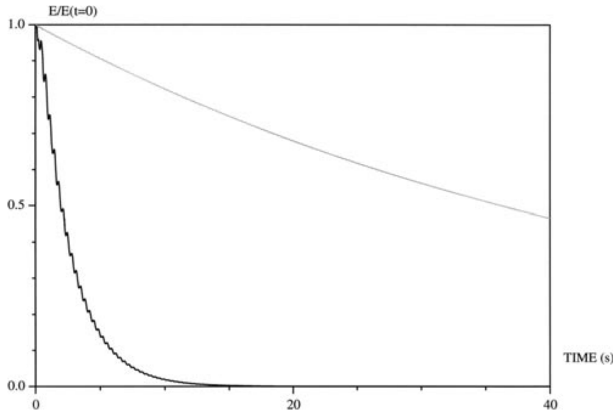

Figure 6. Mechanical energy: release test under closed loop (black line) and open loop (grey line) conditions.

are assumed to be negligible (in order to simplify analytical developments). The output of the sensor and the required input voltage obtained using the two methods (analytical and discrete) are plotted in figures 4 and 5. For each figure, the difference between the results cannot be seen as they are almost identical: the finite element method, using the simple finite beam element, gives the same results as the analytical calculus: it is a first validation of the element.

We also studied the active control of the three-beam structure shown figure 1, equipped with two actuators

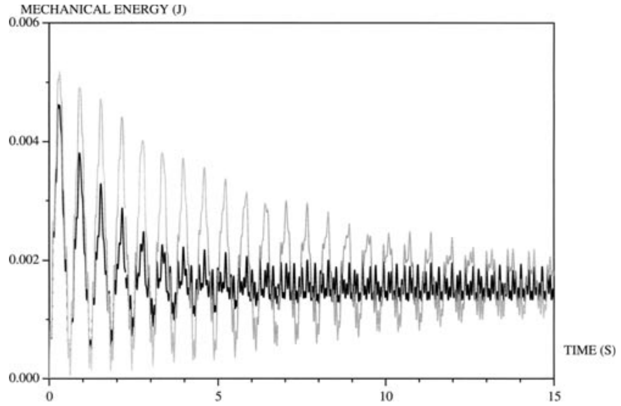

Figure 7. Mechanical energy: sinusoidal test under closed loop (black line) and open loop (grey line) conditions.

and sensors. The geometrical and mechanical properties of the system are detailed in tables 2 and 3. The structure is first subjected to a release test derived from the load $\vec{F}(t) = 0.05\vec{x}$ applied to B_4. Because of the nature of the excitation we take into account the four first eigenmodes. The mechanical energy of the system for the open and the closed loops is plotted in figure 6. Using active control, the mechanical energy vanishes in less than 4 s.

In the same way, the structure is studied when subjected to a persistent external force applied to B_4 and equal to $\vec{F}(t) = 0.6\cos(40t)\vec{x}$. From figure 7, the active control stabilizes the mechanical energy in less than 8 s whereas for the open loop it requires more than 15 s.

These simulations show the efficiency of the active control system used to attenuate vibrations of beam structures, and the interest of a finite element model in this context.

5. Conclusion

In order to simulate the active control of beam structures, we have developed here a finite beam element to model actuators and sensors, taking into account the piezoelectric effect. Using a usual control strategy, but starting from discrete finite element equations, a linear quadratic control method including a state observer has been used to compute the control. Simulation of the active control of a simple cantilever beam validates the developed finite element.

Simulations for a three-beam structure show that active control can be very efficient. However, we can show that this efficiency depends strongly on the location of the sensors and actuators [5, 9]. The simple finite beam element presented here can easily be used to solve this kind of structural optimization problem, minimizing the computation cost and allowing one to investigate many generic problems on rather simple models without loss of generality. The first applications are presented in [12]. With this aim, we are now working to develop a methodology for the determination of the optimal geometries of piezoelectric devices.

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